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GAS-DYNAMIC STRUCTURES IN SUPERSONIC COMBUSTION OF HYDROGEN IN A SYSTEM OF PLANE JETS IN A SUPERSONIC FLOW

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Results of numerical and theoretical studies of supersonic diffusive combustion of a system of plane hydrogen jets in a supersonic air flow are described. It is shown that large-scale vortex structures appear in the mixing zone of the system of hydrogen jets and the cocurrent flow. These vortex structures affect the mechanism of turbulent exchange between the fuel and the oxidizer.

Introduction. Theoretical and experimental studies of hydrogen combustion in supersonic flows involve some difficulties. The theoretical study is complicated by the fact that the laws of combustion at supersonic velocities are determined by the intensity of turbulent-exchange processes, chemical-reaction rates in the flow, and gas-dynamic effects that accompany heat release. Each of these factors may exert a significant effect on the combustion process. It is next to impossible to perform full-scale experiments under laboratory conditions for flow Mach numbers $M > 6$. Therefore, the development of a reliable numerical-theoretical method for analysis of combustion in supersonic flows becomes extremely important.

There are numerous experimental and numerical-theoretical investigations of combustion in supersonic jet flows for $M < 3$ [1–8]. A review of papers published before 1984 can be found in [1]. The influence of turbulence on the rate of chemical reactions with accounting for intermittency was studied in [2–4], where it was found that chemical reactions have many stages and it is necessary to take into account the effect of concentration fluctuations on the rate of chemical transformations [3, 4]. The laws of supersonic combustion taking into account shock waves were studied in [4, 5], and the effect of shock waves on the ignition and mixing processes was shown. It follows from the experimental data of [6] that the mixing of a hydrogen jet with a cocurrent air stream is one of the main conditions of supersonic burning of hydrogen. The processes of mixing and chemical kinetics of supersonic combustion of hydrogen in jet flows were calculated in [7, 8].

Detailed experimental data on the turbulent structure of supersonic shear flows allowed one to study the laws of turbulent mixing [9] and evaluate the applicability of advanced models of turbulence for their description. In particular, weak anisotropy of turbulence in the mixing zone of two supersonic flows was found [9].

The study of the mixing mechanism is of practical interest. The lack of data on the influence of shock waves on the characteristics of turbulence should be noted.

The objective of the present work is to show that the use of a system of parabolized Navier–Stokes equations in combination with the approved models of turbulence and hydrogen-oxidation kinetics offers a better notion on the laws of combustion of a system of plane supersonic hydrogen jets in a cocurrent supersonic air stream.

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Physicomathematical Model of the Flow. The problem of diffusive combustion of a system of plane supersonic hydrogen jets in a cocurrent supersonic air stream is considered.

A hydrogen jet with a supersonic velocity U_1 escapes from plane nozzles of height $2h_1$ into a cocurrent supersonic air stream that moves with a velocity U_2. The gases of the jet and the cocurrent flow have different molecular weights (a mixture of hydrogen and nitrogen in the jet and air in the cocurrent flow). The OX axis is directed along the plane of symmetry of the jet, and the OY axis is perpendicular to it. Since the system of cocurrent jets is periodically repeated, we can take a band of width L limited by the planes of symmetry of the jet and the cocurrent flow and consider the solution in this region, using the conditions of symmetry along the planes OX and AA_1 instead of the rejected part (see [10]).

It is assumed that the flow is supersonic in the entire region, the gas is viscous, heat-conducting, and chemically reacting, and the flow regime is turbulent.

To describe the mean flow, we use a system of parabolized equations of conservation of momentum, energy, matter, and continuity, which is written in the matrix form:

$$\frac{\partial \boldsymbol{F}}{\partial x} + \frac{\partial \boldsymbol{G}}{\partial y} = \frac{\partial}{\partial y}\left(\boldsymbol{S} + \boldsymbol{H}\right) + \boldsymbol{W}. \tag{1}$$

Here $\boldsymbol{F} = \boldsymbol{F}\left[\rho u, \rho u^2 + p, \rho uv, (\rho E + p)u, \rho u C_i\right]$, $\boldsymbol{G} = \boldsymbol{G}\left[\rho v, \rho uv, \rho v^2 + p, (\rho E + p)v, \rho v C_i\right]$, $\boldsymbol{W} = \boldsymbol{W}\left[0, 0, 0, 0, \dot{W}_i\right]$,

$$\boldsymbol{S} = \boldsymbol{S}\Big[0, \mu_{\mathrm{t}} \frac{\partial u}{\partial y}, \frac{4}{3}\mu_{\mathrm{t}} \frac{\partial v}{\partial y}, \gamma \frac{\mu_{\mathrm{t}}}{\mathrm{Pr_t}} \frac{\partial e}{\partial y} + \mu_{\mathrm{t}}\Big(\frac{1}{2}\frac{\partial u^2}{\partial y} + \frac{2}{3}\frac{\partial v^2}{\partial y}\Big), \frac{\mu_{\mathrm{t}}}{\mathrm{Sc_t}} \frac{\partial C_i}{\partial y}\Big],$$

$$\boldsymbol{H} = \boldsymbol{H}\Big[0, 0, 0, \frac{\mu_{\mathrm{t}}}{\mathrm{Sc_t}}\Big(\sum_i h_i \frac{\partial C_i}{\partial y}\Big), 0\Big],$$

u and v are the longitudinal and transverse components of velocity, ρ is the density, p is the pressure, e is the specific internal energy, C_i is the concentration of matters in the mixture, μ_{t} is the vortical turbulent viscosity, and $\mathrm{Pr_t} = \mathrm{Sc_t} = 0.9$ are turbulent analogs of the Prandtl and Schmidt numbers.

All the quantities that enter into system (1) are dimensionless. They are normalized to the velocity U_1, hydrogen density ρ_1, and height h_1. The coordinates x and y are normalized to h_1, u and v to U_1, ρ to ρ_1, p to $\rho_1 U_1^2$, e to U_1^2, and μ_{t} to $\rho_1 U_1 h_1$.

The equation of state for the perfect gas is written in the form

$$p = \rho R T, \tag{2}$$

where $R = R_0 \sum\limits_i \dfrac{C_i}{m_i}$ (m_i is the molecular weight of the ith component of the mixture and R_0 is the universal gas constant).

The total energy is

$$E = \sum_i C_i \int\limits_{T_0}^{T} c_{vi}\, dT + \frac{u^2 + v^2}{2} + \sum_i h_i^0 C_i, \tag{3}$$

where c_v is the specific heat capacity at constant volume.

The Wilke formula [11] was used to calculate the thermophysical properties of the hydrogen–oxygen mixture.

The coefficient of turbulent dynamic viscosity μ_{t} was determined on the basis of the one-parameter k–l_ω model of turbulence:

$$\mu_{\mathrm{t}} = C_\omega \rho l_\omega \sqrt{k}. \tag{4}$$

Here l_ω is the mixing-zone length. The turbulent kinetic energy k is found from the equation

$$\rho u \frac{\partial k}{\partial x} + \rho v \frac{\partial k}{\partial y} = \frac{\partial}{\partial y}\Big(\mu_{\mathrm{t}} \frac{\partial k}{\partial y}\Big) + \mu_{\mathrm{t}}\Big(\frac{\partial u}{\partial y}\Big)^2 + \frac{\mu_{\mathrm{t}}}{\rho\, \mathrm{Sc_t}} \frac{\partial \rho}{\partial y} \frac{\partial k}{\partial y} - \frac{C_d \rho k^{3/2}}{l_\omega}. \tag{5}$$

Generation of the turbulent kinetic energy in Eq. (5), which is caused by the variable density field, is written in accordance with [12]. The constants in the model acquire the following values: $C_\omega = 0.07$ and $C_d = 0.13$.

The following expression is used to determine l_ω in non-self-similar jet flows:

$$l_\omega = \frac{u_{\max} - u_{\min}}{(du/dy)_{\max}}.$$

The burning rate of hydrogen in air is described by a mechanism that takes into account the concentrations of six active components (H, O, OH, H_2O, O_2, and H_2) and inert nitrogen:

$$\begin{array}{lll} H+O_2 \rightleftarrows OH+O, & O+H_2 \rightleftarrows OH+H, & H_2+OH \rightleftarrows H+H_2O, \\ OH+OH \rightleftarrows O+H_2O, & H_2+M \rightleftarrows H+H+M, & H_2O+M \rightleftarrows OH+H+M, \\ OH+M \rightleftarrows O+H+M, & O_2+M \rightleftarrows 2O+M, & H_2+O_2 \rightleftarrows 2OH. \end{array}$$

Detailed information on the rates of these reactions and constants is given in [1].

The effect of intermittency on the magnitude of the averaged rates of elementary chemical reactions was taken into account by using Spiegler's immiscibility model [13]. Though this model is considered to be rather simple, the use of this model leads to better agreement of numerical and experimental results [3].

System (1)–(5) was integrated under the following boundary conditions:

(a) at the entrance cross section $x = 0$:

— in the jet, we have

$$u = 1, \quad v = 0, \quad \rho = 1, \quad e = \frac{C_{v1}}{\mathrm{M}_1^2 R_1 \gamma}, \quad k = C_k u_1^2, \quad C_i = C_{i1}, \tag{6}$$

— in the flow, we have

$$u = \frac{\mathrm{M}_2}{\mathrm{M}_1}\sqrt{\frac{R_2 T_2}{R_1 T_1}}, \quad v = 0, \quad \rho = \frac{T_1 R_1}{n T_2 R_2}, \quad e = \frac{C_{v2} T_2}{\mathrm{M}_1^2 T_1 R_1 \gamma}, \quad k = C_k u_2^2, \quad C_i = C_{i2}; \tag{7}$$

(b) for $x > 0$, $y = 0$, and $y = L$, the conditions of symmetry are set:

$$\frac{\partial u}{\partial y} = \frac{\partial \rho}{\partial y} = \frac{\partial e}{\partial y} = \frac{\partial C_i}{\partial y} = \frac{\partial k}{\partial y} = 0, \qquad v = 0. \tag{8}$$

The boundary conditions in the entrance cross section (6) and (7) are stepwise. They are obtained using the equation of state (2) for convenience of setting the regime flow parameters in computations. The experiments of [9] show that the thickness of the mixing zone of two supersonic parallel shear flows increases very slowly in the streamwise direction. Therefore, the neglect of the boundary-layer thickness in the entrance cross section and setting profiles (6) and (7) do not violate the physics of the phenomenon. In accordance with the physical model of the problem, relations (8) express the conditions of symmetry. Setting other conditions at the upper boundary ($y = L$), for example, the absence of reflection of disturbance waves, defines the problem of supersonic diffusive burning of a single plane hydrogen jet in a cocurrent supersonic flow.

System (1)–(8) was solved numerically. The convective terms $\partial \boldsymbol{G}/\partial y$ with the pressure gradient in the OY direction were approximated taking into account the sign of eigenvalues of the Jacobi matrix $B = \partial \boldsymbol{G}/\partial \boldsymbol{U}$ with the use of Steger and Warming's scheme [14]. The viscous terms were approximated by the central difference. The assumption of a supersonic flow in the OX direction leads to positive values of the Jacobi matrix $A = \partial \boldsymbol{F}/\partial \boldsymbol{U}$, which allows one to use the marching technique for the solution along the longitudinal coordinate. The finite-difference analogs of the equations of conservation of mass, momentum, and energy are solved jointly by the method of matrix sweeping [15]. In the general iterative process, the equation of conservation of matter for each component was solved separately from the basic system by scalar sweeping. The data of [7] were taken into account in approximating the expressions for the rates of elementary chemical reactions. The method of numerical calculation of the system of gas-dynamic equations was tested in [10].

For testing the turbulence model, we solved the problem of mixing of two supersonic flows in accordance with the conditions of experiments conducted on a test facility developed for studying turbulent shear flows of a compressible gas [9]. Based on the experimental data, the growth rate of the layer thickness for the same velocity of mixing flows was represented in the form of the dependence

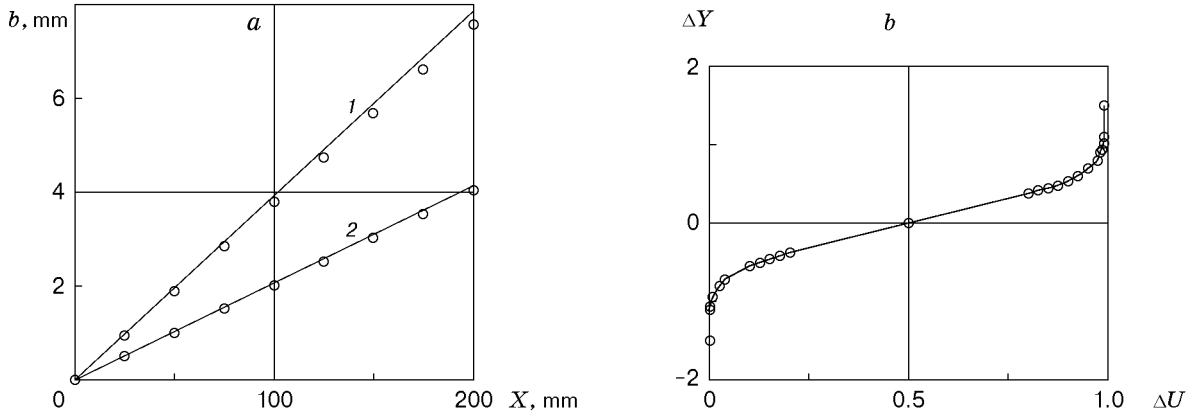

Fig. 1. Mixing-zone width $b(X)$ (a) and self-similar profiles of the longitudinal component of velocity (b): the points refer to experiments and the solid curves refer to calculations for $M_1 = 2.01$, $M_2 = 1.38$, $T_1 = 163$ K, $T_2 = 214$ K, and $n = 1$ (1) and $M_1 = 1.91$, $M_2 = 1.36$, $T_1 = 334$ K, $T_2 = 215$ K, and $n = 1$ (2).

$$\frac{db}{dx}\Big|_{M_r=0} = 0.165\,\frac{(1-r)(1+s^{1/2})}{2(1+rs^{1/2})},$$

where b is the mixing-layer thickness determined as the difference of transverse coordinates, in which $U = U_1 - 0.1(U_1 - U_2)$ and $U = U_2 + 0.1(U_1 - U_2)$, $r = U_2/U_1$ is the ratio of velocities of the mixing flows, $s = \rho_2/\rho_1$ is the ratio of densities of the mixing flows, $M_r = \Delta U/\bar{a}$ is the relative Mach number, and $\bar{a}$ is the arithmetic mean of the velocities of sound in the mixing flows.

The mixing zone of two supersonic air streams was calculated using system (1)–(5). The boundary conditions at the entrance section behind the dividing plate were chosen in accordance with the test conditions of [9], and the conditions of smooth conjugation were used in the flows. The width of the mixing zone $b(X)$ was found from the numerical distributions of the longitudinal component of velocity in each cross section of the flow. As is shown in Fig. 1a, the calculation results are in good agreement with experimental data if the one-parameter k–l_ω model of turbulence is used.

The calculated profiles of the longitudinal component of velocity obtained using the one-parameter k–l_ω model of turbulence were treated in the self-similar variables

$$\Delta U = \frac{U - U_2}{U_1 - U_2}, \qquad \Delta Y = \frac{Y - Y_0}{b(X)},$$

where Y_0 is the transverse coordinate corresponding to the position of the dividing plate.

The profiles were treated in self-similar variables for three regimes of evolution of the mixing zone of two supersonic flows. For each regime, longitudinal velocity profiles were constructed in the cross sections $X = 100$, 125, 150, 175, and 200 mm from the beginning of the mixing zone. The mixing-zone width $b(X)$ in each regime corresponds to the calculated data. It is shown that the self-similar profiles of the longitudinal component of velocity are in good agreement with each other and with the experimental data (Fig. 1b).

Thus, the calculations and their comparison with experimental data indicate that it is possible to use the one-parameter k–l_ω model of turbulence to analyze the features of supersonic jet flows.

To verify the generalized mathematical model of hydrogen combustion, the numerical data were compared with the test results of [2]. Both in calculations and experiments, the beginning of ignition has the coordinate $x = 45$. The change in pressure in the downstream direction and the distributions of concentrations of hydrogen, oxygen, and water-vapor molecules, and also the temperature in the exit cross section of the channel are in good agreement with the experiment. The pressure in the exit cross section ($x = 89$) in the experiment is 20% higher than the pressure at the ignition point.

Discussion of Calculation Results. The main regime parameters of the flow are the nozzle pressure ratio $n = p_1/p_2$, the Mach numbers of the jet M_1 and the flow M_2, the initial concentration of hydrogen in the jet $C^0_{H_2}$, the temperatures of the jet T_1 and the flow T_2, the air-to-fuel ratio g, and the ratio of the flow width to the height of the exit cross section of the jet $d = H/h$.

Detailed fields of pressure, temperature, velocity, and concentration of matters that participate in chemical reactions were obtained as a result of calculations with the use of the one-parameter k–l_ω model of turbulence.

Figure 2 shows the calculation results for combustion of a supersonic hydrogen jet in a cocurrent supersonic air stream. Figure 2a shows the structure of the gas-dynamic section, the regions of expansion and compression, the interaction of disturbance waves with formation of shock waves, and their reflection from the axial and boundary planes of the flow. The ignition of the hydrogen jet occurs in the expansion region (at a distance $x = 30$), which involves propagation of disturbance waves. The ignition does not begin at the nozzle edge; its delay is caused by conditions of turbulent mixing and kinetics of chemical reactions of hydrogen oxidation. The pressure increase in combustion regions increases the intensity of disturbance waves in the downstream direction. Interaction with the pressure field of nonisobaric exhaustion of the supersonic jet forms a Y-shaped structure of disturbance waves and leads to redistribution of the structure of the gas-dynamic section (Fig. 2a).

It follows from the calculations that the maximum concentration of water vapor is reached at the shock-wave front, which indicates a significant influence of shock waves on diffusive combustion of hydrogen (Fig. 2b). The shock waves increase the intensity of mixing of the fuel and oxidizer and the temperature of the mixture at the places where shock waves pass. Taking into account the concentrations of reagents yields a finite thickness of the reaction region. The gas-dynamic, thermal, and kinetic characteristics are such that the reaction region in the regime considered is enclosed within the section $x = 840$.

A dramatic change in temperature occurs on shock waves. The temperature increase on shock waves favors hydrogen-oxidation reactions due to the Arrhenius dependence of the reaction rate on temperature. In turn, hydrogen combustion, being the reason for intense heat release, leads to an increase in temperature and pressure in combustion regions.

The isotachs of the longitudinal velocity illustrate the mixing of the hydrogen jet and the cocurrent air stream (Fig. 2c). The ignition of the hydrogen jet occurs in the zone of intense exchange of the hydrogen jet and the cocurrent air stream, and the wave structure of the flow affects the mixing and combustion processes.

Periodic large-scale vortex structures appear in the mixing zone of the jet and the flow (Fig. 2c). Their appearance is caused not only by the wave structure of the gas-dynamic section but also by the different composition of the jet and flow gases. If the composition of the jet and flow gases is identical, large-scale cellular structures on the isotachs of the longitudinal velocity are not observed, though the densities of the jet and flow gases are different because of the compressibility and nonisobaric exhaustion. The calculations show that these large-scale structures appear if the jet gas contains hydrogen, which increases the difference in the jet and flow gases. The action of a system of oblique shocks on the velocity field leads to deceleration of the gas flow behind the shock and acceleration in front of it, and causes transverse overflow of the gas. The transverse flows are positive in the region of flow deceleration and negative in the region of flow acceleration. Increasing the difference in density of the mixing gases of the jet and the flow, the difference in the composition of the jet and flow increases the intensity of convective flows in the mixing zone and leads to the formation of cellular structures in the field of the longitudinal velocity.

The vorticity field corresponding to the distribution of velocity components is plotted in Fig. 2d. In accordance with the position of large-scale structures in the distributions of the longitudinal velocity, one can see the changes in the isolines of the velocity vortex.

An increase in the nozzle-pressure ratio, other conditions being equal, leads to an increase in the intensity of disturbance waves, a decrease in the jet–flow mixing rate because of the exhaustion of a denser mixture, and an ignition delay. Large-scale vortices appear in the mixing zone.

An increase in the initial concentration of hydrogen in the jet ($\mathrm{M}_1 = 1.15$, $\mathrm{M}_2 = 2.63$, $n = 2$, $T_1 = 454$ K, $T_2 = 1270$ K, and the initial compositions of the gas are $C^0_{\mathrm{H}_2} = 0.4$ and $C^0_{\mathrm{N}_2} = 0.6$ in the jet and $C^0_{\mathrm{O}_2} = 0.265$, $C^0_{\mathrm{N}_2} = 0.675$, and $C^0_{\mathrm{H}_2\mathrm{O}} = 0.06$ in the flow) leads to a decrease in the intensity of disturbance waves, the length of the gas-dynamic section, and the jet–flow mixing rate. At the same time, the difference

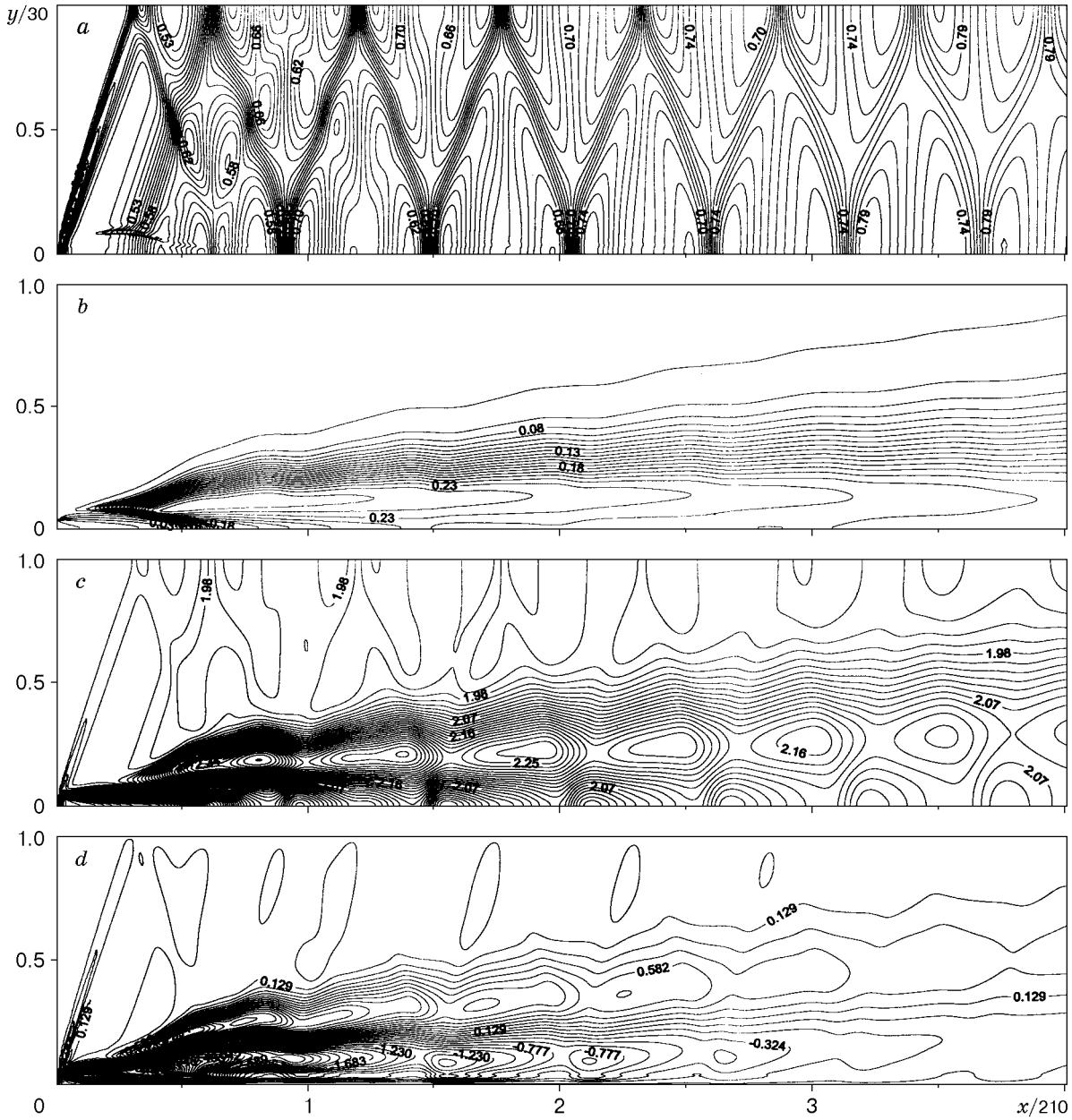

Fig. 2. Isobars (a), isolines of the water-vapor concentration (b), isotachs (c), and isolines of vorticity (d) for supersonic diffusive combustion of a system of plane hydrogen jets in a cocurrent flow ($M_1 = 1.15$, $M_2 = 2.63$, $n = 2$, $T_1 = 454$ K, $T_2 = 1270$ K, and the initial compositions of the gas are $C^0_{H_2} = 0.2$ and $C^0_{N_2} = 0.8$ in the jet and $C^0_{O_2} = 0.265$, $C^0_{N_2} = 0.675$, and $C^0_{H_2O} = 0.06$ in the flow).

in density of the jet and the flow increases, which leads to an increase in the intensity of transverse flows in the mixing zone and the size of large-scale structures on the isotachs of the longitudinal velocity. An increase in heat-release intensity due to chemical reactions leads to an increase in temperature and, thus, to a greater difference in density in the mixing zone.

With increasing difference in the Mach numbers of the jet and the flow ($M_1 = 2$ and $M_2 = 6$), the size of macroscopic structures on the isotachs of the longitudinal velocity decreases. Obviously, the growth in the flow Mach number causes compression of the jet and restricts the development of transverse flows and the size of large-scale structures on the isotachs of the longitudinal velocity.

The results of numerical-theoretical investigations of gas-dynamic laws of supersonic combustion of hydrogen allow the following conclusions.

1. The solution of the system of parabolized Navier–Stokes equations together with the equations of chemical kinetics of hydrogen oxidation and the one-parameter model of turbulence by the method of flux-vector splitting allows one to obtain detailed data on the gas-dynamic, thermal, and geometric structure of the flow in a wide range of nozzle pressure ratios, cocurrency, and initial concentration of hydrogen.

2. Disturbance waves initiated by hydrogen combustion interact with disturbance waves caused by nonisobaric exhaustion of the jet and form a new system of wave structures on the gas-dynamic section in the case of diffusive supersonic combustion of a system of plane hydrogen jets in a cocurrent flow.

3. The numerical data show that turbulent mixing of the hydrogen jet with the cocurrent flow occurs in large-scale structures, which causes an increase in jet–flow mixing intensity and in the combustion efficiency.

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